Exam "Toepassingen van de Quantum-Fysica" Groningen, 10-11-2000

The book of Haken and Wolf may be used during the examination

please: **each problem on a separate sheet!** (to allow correction of each problem by a different person)

Problem 1. Nuclear charges shielded by an electronic core

Neon has the atomic number $Z=10$.

a) give the electronic configuration of the ground state as well as the total orbital angular momentum **L** and the total spin **S**

b) Neon has an ionization potential (i.e. the binding energy of the most loosely bound electron) of 21.56 eV. Calculate from this an effective charge Z_{eff} "seen" by the most loosely bound electron (make use of the fact that the ionization potential of the H atom is 13.6 eV

We now consider excited states of Ne with a configuration $(\ldots 2p^5 3s)$, $(\ldots 2p^5 4s)$, $(\ldots 2p^5 5s)$ etc

c) what are the possible values for the total orbital angular momentum, the total spin and the total angular momentum of these configurations (make use of the fact that one vacancy in an otherwise completely filled shell can be regarded as a single electron in an otherwise completely empty shell!)

One of the states with configuration $(\ldots 2p^5 3s)$ can be excited from the ground state by irradiating the Ne with light at a wavelength $\lambda = 73.6$ nm.

d) Assume now that the 3s electron "sees" a Ne^+ core with an effective charge of $Z_{eff} = 1$ and calculate the quantum defect Δn which gives the "correct" result for the binding energy of the 3s electron.

e) Assume that the same quantum defect is valid for the 4s and 5s electron and from this calculate the wavelengths of light, necessary to excite from the groundstate the corresponding states with $(\ldots 2p^5 4s)$, $(\ldots 2p^5 5s)$ configuration.

Problem2 Zeeman slower

We want to decelerate an atomic beam consisting of metastable He($1s2s\ ^3S_1$) atoms by means of a resonant laser beam coming from the opposite direction using the transition He($1s2s\ ^3S_1 \rightarrow 1s2p\ ^3P_2$) at a wavelength $\lambda = 1083$ nm. The He($2\ ^3P_2$) state has a radiative lifetime $\tau = 100$ ns. Suppose that all the metastable He($2\ ^3S_1$) atoms leave the source with the same velocity $v_o = 1000\ ms^{-1}$ and that the mass of the He atoms equals $M = 4$ amu. Assume that the laser intensity is so strong that a decaying $^3P_2$ atom is immediately excited again.

a) What is a metastable state and why is the He($1s2s\ ^3S_1$) state metastable ?

b) Explain in qualitative terms why an atomic beam can be brought to rest in this way.

c) Calculate the time needed to stop the atoms and the total length they travel before brought to rest assuming that the laser stays resonant with the atomic transition.

Unfortunately the stopping atoms experience a changing Doppler shift $\Delta\nu/\nu = v/c$ with respect to the fixed laser frequency.

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d) Calculate the natural line width of the atomic transition and compare this with the Doppler shift of atoms moving with a speed of v_o= 1000 ms^{-1}.

We want to keep the laser frequency constant and resonant with atoms at rest in a field-free region. In order to do so we use a position-dependent magnetic field shaped in such a way that the Zeeman shift cancels the Doppler shift along the entire atomic trajectory. We use circularly polarized light.

e) Calculate the necessary position dependency of the magnetic-field strength $B(z)$ as a function of the position z along the atomic beam axis.
(Hint: determine first how the atomic velocity v depends on the position z.)

f) Could you also use the He($1s2s$ $^1S_o \rightarrow 1s2p$ $^1P_1$) transition to stop metastable He atoms ? Why ? Or why not?

Problem3 — *Fine- and Hyperfinestructure in K(4p)*

In the potassium atom, spin-orbit splitting between the states $4P_{1/2}$ and $4P_{3/2}$ leads to a wavelength difference of $\Delta\lambda$ = 3.4 nm for the first line pair of the primary series, with λ = 766.7 nm for the shorter wavelength.

a) Derive a formula which gives the relation between the fine structure constant a and the magnetic field experienced by the valence electron spin due to the electronic orbital motion.
b) From $\Delta\lambda$ = 3.4 nm calculate the fine structure constant a and the magnetic field B_l
c) Consider the isotope ^{40}K with a nuclear spin $I=4$. Indicate the hyperfinesplitting of the K(4p) states
d) The nuclear magnetic moment of ^{40}K is $\mu_I = -1.298\ \mu_N$. Calculate the resulting hyperfine splitting for the $4P_{1/2}$ state.

Problem 4. — *Perturbation theory*

We consider a one-dimensional potential well with infinitely high walls and a flat bottom given by V=0 for $|x|<a$ and $V=\infty$ for $|x|>a$.

a) Give the wave functions u_0 and u_1 for the two lowest eigen states of a particle in this well.
b) Calculate the energy eigen values E_0 and E_1 for these two states.

Now we consider a perturbation of the potential given by V_p=A cos(πx/2a) for $|x|<a$, with $A \ll E_0$.

c) Give the Schrödinger equation for this situation
d) Use first order perturbation theory to calculate the new energies